COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-63/21
12. 4. 1963.

A. Fujii and L. Holloway: A UNITARY SYMMETRY MODEL FOR PHOTO PRODUCTION. -

Nota interna : n. 192

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In this note we would like to consider photoproduction of pseudoscalar mesons from the viewpoint of Unitary Symmetry. The underlying basis of these considerations is the assignment of the known (or conjectured) particles and resonances to irreducible representations of a group which satisfies the Lie Algebra. Once this assignment has been made the problem is reduced, as for as the group theoretical part is concerned, to finding the irreducible representations which are contained in the direct product space of two representations. Results are then obtained in the form of relations between the amplitudes for various photoproduction processes.

Once those relations have been obtained however, one has to consider how realistic they are from the standpoint of physical reality. In other words, it is clear that the mesons and baryons do not correspond exactly to a representation of a Lie Group due to their mass differences. So the problem arises of to what extent and in what energy regions are the predictions of the unitary symmetry models useful. Perhaps at very high energies, when the mass differences can be neglected, the relations become exact. It is certainly true that at low energies these are grave difficulties. For example, as we will show later, an attractive (from the viewpoint of unitary symmetry) model for photoproduction consists of considering the two step process $\gamma + N \rightarrow N^* \longrightarrow B + m$, where the N^* belongs to a ten-fold representation and the final state, B + m, can be either $N\pi$ or ΣK. This model contains the T = 3/2 resonance in π photoproduction but also predicts a T = 3/2 resonance in the ΣK production cross section at the N mass. However, this resonance is 500 MeV below threshold. In this particular case an attempt to patch up the unitary symmetry prediction by phase space corrections is futile.

We begin our discussion by reviewing some of the properties of Lie Groups. Of the four simple compact Lie Groups of rank two, the one called SU3 lends itself to simple correspondences with the known particles, namely the Sakata Model and the eight-fold way of Gell-Mann[1] and Ne'eman[2]. The Sakata Model is based on the following assignment:

$$D^8(1,0) \longleftrightarrow (p\, n\, \Lambda)$$
$$D^8(1,1) \longleftrightarrow (\pi\, K\, \eta),$$

and the eight-fold way is based on the following assignment:

$$D^8(1,1) \longleftrightarrow (N\, \Lambda\, \Sigma\, \Xi)$$
$$D^8(1,1) \longleftrightarrow (\pi\, K\, \eta).$$

$D^n(\lambda_1, \lambda_2)$ denotes an irreducible representation of SU_3 where n is the dimensionality of the representation and the highest weight is $\lambda_1\, m_1 + \lambda_2 . m_2$ where m_1 and m_2 are the two fundamental dominant weights of the groups[3].

The weight diagrams for $D^3(1,0)$ and $D^8(1,1)$ are given in figures 1 and 2. The two axes of the weight diagrams correspond to the eigenvalues of the two mutually commuting operators of the group and can be identified with hypercharge Y and the third component of isotopic spin T_3.

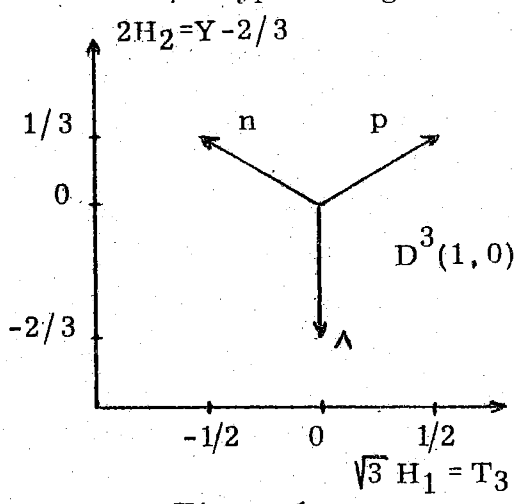

Figure 1

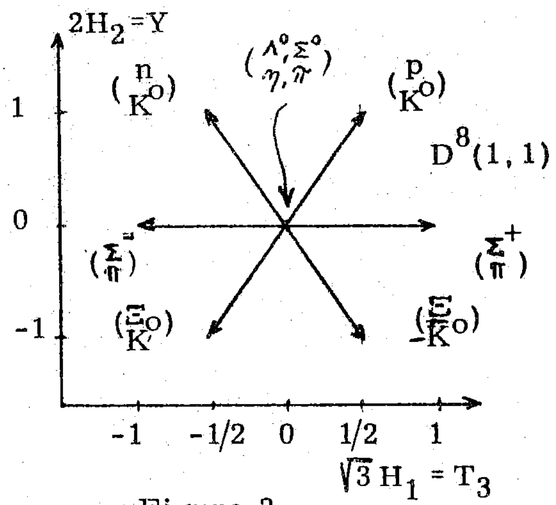

Figure 2

From the theory of Lie Algebras it is known that one can write the commutation relations of the group generators in the form:

$$\left[F_m, F_n\right] = i\, f_{mne}\, F_e$$

where the f's are real and completely antisymmetric.

We observe that the charge Q can be written as:

$$Q = T_3 + Y/2$$

For the eight-fold way, where one makes the following correspondences:

$$\sqrt{3}\, H_1 = F_3 = T_3$$

and

$$2H_2 = 2/\sqrt{3}\, F_8 = Y;$$

the electromagnetic current will be given by

$$i_{em} = j_3 + 1/\sqrt{3}\, j_8$$

The currents j_m satisfy

$$\left[F_m, j_n\right] = i\, f_{mnl}\, j_l$$

i. e. they belong to the regular eight-dimensional representation.

The transformation properties of the initial state can now be written as

$$\langle N | j \sim \langle N \pi^o | + 1/\sqrt{3} \langle N \eta |$$

where "$\sim$" reads "transforms like". From now on, as far as the eight--fold way is concerned, we consider the photon as ($\pi^o + 1/\sqrt{3}\, \eta$).

We now proceed to construct the irreducible representations of the 8 ⊗ 8 product space of a baryon and a meson. We use the rules of BDFL in constructing the state vectors:

a) Select the unique state with the highest weight (The highest weight is the sum of the highest weights of the product representations).

b) Use the raising and lowering operators $E_{\pm\alpha}$ to generate the remaining state vectors in the representation (A schmidt orthogonalization procedure may be necessary). The weights of the generated states can be determined with the use of the operators H_1 and H_2.

c). After the repeated application of the E's exhaust the highest weight representation, select the state with the highest weight from the remaining possibilities and continue step b) untill all of the irreducible representations are constructed.

The following of the above rules for baryon ⊗ meson yields Table I for the basis vectors. In the state vector $\langle N; Y\, T|$, N corresponds to the dimensionality of the representation and Y and T are the hypercharge and I-spin.

We are now in a position to form the relation between the various photoproduction processes. We are interested in amplitudes of the form $\langle N | j | BM \rangle$. If we write

$$| BM \rangle = \sum_{iT} a_{iT} | Ni; Y, T \rangle$$

we can find the a's from Table I. Furthermore we need the combinations $\langle p\,\pi^o| + 1/\sqrt{3}\langle p\,\eta|$ and $\langle n\,\pi^o| + 1/\sqrt{3}\langle n\,\eta|$ which correspond to the transformation properties of $\langle p|j$ and $\langle n|j$ respectively:

$$\langle p|j \sim \sum_{iT} b_{iT} \langle N_i; Y, T|$$

The a and b coefficients are shown in Table II. Then

$$\langle p|j|BM\rangle = \sum_{iT} A_i \; b_{iT}\, a_{iT}$$

and Table III results. The a's are the group amplitudes and are eight in number; A_1, A_8, $A_{8'}$, $A_{8'8}$, $A_{8\,8'}$, A_{10}, $A_{\overline{10}}$, and A_{27}. The amplitudes $A_{88'}$ and $A_{8'8}$ exist because of the equivalence between the 8 and 8' representations in the direct product $8 \otimes 8 = 1 + 8 + 8' + 10 + \overline{10} + 27$. The 10 and $\overline{10}$ representations are inequivalent.

The following relations can be written for the photoproduction amplitudes (The symbol $A(n\pi^+)$ denotes $A(\gamma + p \rightarrow n + \pi^+)$ etc.)

1) $\sqrt{2}\, A(n\,\pi^+) + A(\Sigma^o K^+) = \sqrt{3}\, A(\Lambda K^+)$

2) $\sqrt{2}\, A(\Sigma^+ K^o) + A(p\,\pi^o) = \sqrt{3}\, A(p\,\eta)$

3) $A(n\,\pi^o) - \sqrt{3}\, A(n\eta) = \sqrt{3}\, A(\Lambda K^o) - A(\Sigma^o K^o)$

4) $A(\Sigma^o K^o) + 1/\sqrt{2}\, A(\Sigma^- K^+) = A(\Sigma^o K^+) + 1/\sqrt{2} A(\Sigma^+ K^o)$

5) $A(n\,\pi^o) + 1/\sqrt{2}\, A(p\,\pi^-) = A(p\,\pi^o) + 1/\sqrt{2} A(n\,\pi^+)$

The relations 1) and 2) yield the following triangle inequalities for the cross sections:

$$2\sigma_{n\pi^+} \leq \sigma_{\Sigma^o K^+} + 3\,\sigma_{\Lambda K^o}$$

$$\sigma_{p\pi^o} \leq 2\sigma_{\Sigma^+ K^o} + 3\sigma_{p\eta}$$

One can, however, obtain stronger predictions if one of the group amplitudes becomes dominant, e. g. through a resonance. For instance the 3/2, 3/2 resonance in the pion-nucleon system can be considered as a member of D^{10}. Then with A_{10} dominant the cross sections at resonance become:

$$\sigma_{p\pi^o} = 2\sigma_{n\pi^+} = 2\sigma_{\Sigma^+ K^o} = \sigma_{\Sigma^o K^+}$$

$$\sigma_{p\eta} = \sigma_{\Lambda K^+} = 0$$

This model fails, at least for the ΣK part because the 3/2, 3/2 resonance is below the physical threshold.

If the A_8 amplitude becomes dominant, for instance at one of the higher nucleon resonances, then the following relations can be written:

TABLE I

$$|27;2,1\rangle^{++} = PK^+$$

$$|27;1,3/2\rangle^{++} = 1/\sqrt{2}\,(\Sigma^+K^+ + P\pi^+)$$

$$|27;1,1/2\rangle^{+} = -\sqrt{3/20}\,\left[1/3(P\pi^0-n\pi^+)+1/3(\Sigma^0K^+-\sqrt{2}\,\Sigma^+K^0)+\sqrt{3}(P\eta+\Lambda K^+)\right]$$

$$|27;0,2\rangle^{++} = -\Sigma^+\pi^+$$

$$|27;0,1\rangle^{+} = -1/\sqrt{10}\,\left[\sqrt{2}(\Xi^0K^+-P\bar{K}^0)+\sqrt{3}\,(\Sigma^+\eta+\Lambda\pi^+)\right]$$

$$|27;0,0\rangle^{0} = 1/\sqrt{120}\;\;(\Sigma^0\pi^0-\Sigma^+\pi^--\Sigma^-\pi^+)+3(n\bar{K}^0+PK^-)+3(\Xi^-K^+-\Xi^0K^0)+9\Lambda\eta]$$

$$|27;-1,1/2\rangle^{0} = -\sqrt{3/20}\,\left[\sqrt{3}(\Xi^0\eta-\Lambda\bar{K}^0)+1/3\,(-\Xi^0\pi^0+\sqrt{2}\,\Xi^-\pi^+)+1/3\,(\Sigma^0\bar{K}^0+\sqrt{2}\,\Sigma^+K^-)\right]$$

$$|27;-1,3/2\rangle^{+} = 1/\sqrt{2}\,\left[-\Sigma^+\bar{K}^0+\Xi^0\pi^+\right]$$

$$|27;-2,1\rangle^{0} = -\Xi^0\bar{K}^0$$

$$|10;1,3/2\rangle^{++} = -1/\sqrt{2}\,(P\pi^+ - \Sigma^+K^+)$$

$$|10;0,1\rangle^{+} = -1/2\sqrt{3}\,\left[(\Sigma^0\pi^+-\Sigma^+\pi^0)+\sqrt{3}\,(\Lambda\pi^+-\Sigma^+\eta)-\sqrt{2}\,(P\bar{K}^0+\Xi^0K^+)\right]$$

$$|10;-1,1/2\rangle^{0} = 1/\sqrt{12}\,\left[(\Sigma^0\bar{K}^0+\sqrt{2}\,\Sigma^+K^-)+(\Xi^0\pi^0-\sqrt{2}\,\Xi^-\pi^+)+\sqrt{3}\,(\Lambda\bar{K}^0+\Xi^0\eta)\right]$$

$$|10;-2,0\rangle^{-} = 1/\sqrt{2}\,\left[\Xi^-\bar{K}^0+\Xi^0K^-\right]$$

$$|\overline{10};2,0\rangle^{+} = 1/\sqrt{2}\,(PK^0 - nK^+)$$

$$|\overline{10};1,1/2\rangle^{+} = -1/\sqrt{12}\,\left[(P\pi^0-\sqrt{2}\,n\pi^+)+(-\Sigma^0K^++\sqrt{2}\,\Sigma^+K^0)+\sqrt{3}\,(\Lambda K^+-P\eta)\right]$$

$$|\overline{10};0,1\rangle^{+} = -1/\sqrt{12}\,\left[(-\Sigma^0\pi^++\Sigma^+\pi^0)+\sqrt{2}\,(\Xi^0K^++P\bar{K}^0)+\sqrt{3}\,(\Lambda\pi^+-\Sigma^+\eta)\right]$$

$$|\overline{10};-1,3/2\rangle^{+} = -1/\sqrt{2}\,(\Xi^0\pi^+ + \Sigma^+\bar{K}^0)$$

$$|8;1,1/2\rangle^{+} = \sqrt{3/20}\left[(\sqrt{2}\,\Sigma^+K^0-\Sigma^0K^+)+(\sqrt{2}\,n\pi^+-P\pi^0)+1/\sqrt{3}\,(\Lambda K^++P\eta)\right]$$

$$|8;0,1\rangle^{+} = \sqrt{3/10}\left[\sqrt{2/3}\,(\Sigma^+\eta+\Lambda\pi^+)+(P\bar{K}^0-\Xi^0K^+)\right]$$

$$|8;0,0\rangle^{0} = 1/\sqrt{5}\;\;(\Sigma^-\pi^++\Sigma^+\pi^--\Sigma^0\pi^0+\Lambda\eta)-1/2\,(PK^-+n\bar{K}^0+\Xi^-K^+-\Xi^0K^0)]$$

$$|8;-1,1/2\rangle^{+} = \sqrt{3/20}\,\left[1/\sqrt{3}\,(\Lambda\bar{K}^0-\Xi^0\eta)+(-\Xi^0\pi^0+\sqrt{2}\,\Xi^-\pi^+)+(\sqrt{2}\,\Sigma^+K^-+\Sigma^0\bar{K}^0)\right]$$

$$|8';1,1/2\rangle^{+} = 1/\sqrt{12}\,\left[(P\pi^0-\sqrt{2}\,n\pi^+)+(\sqrt{2}\,\Sigma^+K^0-\Sigma^0K^+)+\sqrt{3}\,(P\eta-\Lambda K^+)\right]$$

$$|8';0,1\rangle^{+} = 1/\sqrt{6}\,\left[\sqrt{2}\,(\Sigma^0\pi^+-\Sigma^+\pi^0)+(P\bar{K}^0+\Xi^0K^+)\right]$$

$$|8';0,0\rangle^{0} = 1/2\,\left[PK^-+n\bar{K}^0+\Xi^0K^0-\Xi^-K^+\right]$$

$$|8';-1,1/2\rangle^{+} = 1/\sqrt{12}\,\left[(\sqrt{2}\,\Xi^-\pi^+-\Xi^0\pi^0)+\sqrt{3}\,(\Xi^0\eta+\Lambda\bar{K}^0)-(\sqrt{2}\,\Sigma^+K^-+\Sigma^0\bar{K}^0)\right]$$

TABLE 2

	$\|27;\ 1,1/2\rangle$	$\|27;\ 1,3/2\rangle$	$\|\overline{10};\ 1,1/2\rangle$	$\|10;\ 1,3/2\rangle$	$\|8;\ 1,3/2\rangle$	$\|8';\ 1,1/2\rangle$
$\|p\pi^0\rangle$	$-1/\sqrt{60}$	$1/\sqrt{3}$	$-1/\sqrt{12}$	$-1/\sqrt{3}$	$-\sqrt{3/20}$	$1/\sqrt{12}$
$\|p\eta\rangle$	$-3/\sqrt{20}$		$1/2$		$1/\sqrt{20}$	$1/2$
$\|n\pi^+\rangle$	$1/\sqrt{30}$	$1/\sqrt{6}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$\sqrt{3/10}$	$-1/\sqrt{6}$
$\|\Sigma^+K^0\rangle$	$1/\sqrt{30}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	$\sqrt{3/10}$	$1/\sqrt{6}$
$\|\Sigma^0K^+\rangle$	$-1/\sqrt{60}$	$1/\sqrt{3}$	$1/\sqrt{12}$	$1/\sqrt{3}$	$-\sqrt{3/20}$	$-1/\sqrt{12}$
$\|\Lambda K^+\rangle$	$-3/\sqrt{20}$		$-1/2$		$1/\sqrt{20}$	$-1/2$
$\langle p\,\|\,j$	$-2/\sqrt{15}$	$1/\sqrt{3}$		$-1/\sqrt{3}$	$-1/\sqrt{15}$	$1/\sqrt{3}$
$\|n\,\pi^0\rangle$	$2/\sqrt{15}$	$1/\sqrt{3}$	$1/\sqrt{12}$	$-1/\sqrt{3}$	$\sqrt{3/20}$	$-1/\sqrt{12}$
$\|p\,\pi^-\rangle$	$-1/\sqrt{30}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$-1/\sqrt{6}$	$-\sqrt{3/10}$	$1/\sqrt{6}$
$\|n\,\eta\rangle$	$-3/\sqrt{20}$		$1/2$		$1/\sqrt{20}$	$1/2$
$\|\Sigma^0\,K^0\rangle$	$1/2\sqrt{15}$	$1/\sqrt{3}$	$-1/\sqrt{12}$	$1/\sqrt{3}$	$\sqrt{3/20}$	$1/\sqrt{12}$
$\|\Sigma^-\,K^+\rangle$	$-1/\sqrt{30}$	$1/\sqrt{6}$	$1/\sqrt{6}$	$1/\sqrt{6}$	$-\sqrt{3/20}$	$-1/\sqrt{6}$
$\|\,\Lambda\,K^0\rangle$	$-3/\sqrt{20}$		$-1/2$		$1/\sqrt{20}$	$-1/2$
$\langle n\,\|\,j$	$-1/\sqrt{15}$	$1/\sqrt{3}$	$1/\sqrt{3}$	$-1/\sqrt{3}$	$2/\sqrt{15}$	

TABLE 3

	A_{27}	$A_{\overline{10}}$	A_{10}	A_8	$A_{8'}$	$A_{8\,8'}$	$A_{8'8}$
$\langle p\,\|\,j\,\|\,p\pi^0\rangle$	$2/5$		$1/3$	$1/10$	$1/6$	$-1/6\sqrt{5}$	$-1/2\sqrt{5}$
$\langle p\,\|\,j\,\|\,n\pi^+\rangle$	$\sqrt{2}/10$		$\sqrt{2}/6$	$-\sqrt{2}/10$	$-\sqrt{2}/6$	$1/2\sqrt{10}$	$1/\sqrt{10}$
$\langle p\,\|\,j\,\|\,p\eta\rangle$	$\sqrt{3}/5$			$-1/10\sqrt{3}$	$\sqrt{3}/6$	$-1/2\sqrt{15}$	$1/2\sqrt{15}$
$\langle p\,\|\,j\,\|\,\Sigma^+K^0\rangle$	$\sqrt{2}/10$		$-\sqrt{2}/6$	$-\sqrt{2}/10$	$\sqrt{2}/6$	$-1/3\sqrt{10}$	$1/\sqrt{10}$
$\langle p\,\|\,j\,\|\,\Sigma^0K^+\rangle$	$2/5$		$-1/3$	$1/10$	$-1/6$	$1/6\sqrt{5}$	$-1/2\sqrt{5}$
$\langle p\,\|\,j\,\|\,\Lambda K^+\rangle$	$\sqrt{3}/5$			$-1/10\sqrt{3}$	$-\sqrt{3}/6$	$1/2\sqrt{15}$	$1/2\sqrt{15}$
$\langle n\,\|\,j\,\|\,n\,\pi^0\rangle$	$3/10$	$1/6$	$1/3$	$1/5$			
$\langle n\,\|\,j\,\|\,p\,\pi^-\rangle$	$\sqrt{2}/5$	$-\sqrt{2}/6$	$\sqrt{2}/6$	$-\sqrt{2}/5$			
$\langle n\,\|\,j\,\|\,n\,\eta\rangle$	$\sqrt{3}/10$	$\sqrt{3}/6$		$\sqrt{3}/15$			
$\langle n\,\|\,j\,\|\,\Sigma^0K^0\rangle$	$3/10$	$-1/6$	$-1/3$	$1/5$			
$\langle n\,\|\,j\,\|\,\Sigma^-K^+\rangle$	$\sqrt{2}/5$	$\sqrt{2}/6$	$-\sqrt{2}/6$	$-\sqrt{2}/5$			
$\langle n\,\|\,j\,\|\,\Lambda\,K^0\rangle$	$\sqrt{3}/10$	$-\sqrt{3}/6$		$\sqrt{3}/15$			

$$\sigma_{p\pi^o} = 1/2\ \sigma_{n\pi^+} = 3\ \sigma_{p\eta} = 1/2\ \sigma_{\Sigma^+K^o} = \sigma_{\Sigma^oK^+} = 3\ \sigma_{\Lambda K^+}$$

One can make an attempt to correct for phase space differences in the cross sections. As an example let us consider η photoproduction cross section relative to that of the π^o. If the A_8 amplitude is dominant Table 4 gives the expected ratio of $\sigma_{p\eta}/\sigma_{p\pi^o}$ corrected for phase space.

TABLE 4

E_γ(MeV)	S	P	D	F
950	-22	-09	-04	-017
1000	-23	-11	-05	-025
1050	-25	-13	-07	-041

Let us now find the corresponding relations for the Sakata Model. If we note that the phenomenological interaction Lagrangian must be invariant under rotation in the unitary space then it must be of the form:

$$L' \sim a_1\ \mathrm{tr}\left[\bar{B}\,\bar{M}\,Q\,B\right] + a_2\ \mathrm{tr}\left[\bar{B}\,Q\,\bar{M}\,B\right] + a_3\mathrm{tr}\left[\bar{B}B\right]\mathrm{tr}\left[\bar{M}Q\right]$$

where $B = \begin{pmatrix} p \\ n \\ \Lambda \end{pmatrix}$

$$M = \begin{pmatrix} 1/\sqrt{6}\,\eta + 1/\sqrt{2}\,\pi^o & \pi^+ & K^+ \\ \pi^- & 1/\sqrt{6} - 1/\sqrt{2}\,\pi^o & K^o \\ K^- & \bar{K}^o & -2/\sqrt{6}\,\eta \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The a's in the above equation are linearly realted to the irreducible group amplitudes. Performing the indicated traces enables us to construct Table 5, from which we obtain the following relations for the photoproduction amplitudes:

$$\sqrt{3}\,A(p\,\eta) = A(p\,\pi^o)$$

$$A(n\,\pi^+) = A(\Lambda\,K^+)$$

$$A(n\,\pi^+) + A(p\,\pi^-) = \sqrt{2}\,A(p\,\pi^o) - \sqrt{2}\,A(n\,\pi^o)$$

$$\sqrt{3}\,A(n\,\eta) = A(n\,\pi^o)$$

$$A(\Lambda\,K^o) = 0$$

TABLE 5

	a_1	a_2	a_3
$\langle p \mid j \mid p\,\pi^o \rangle$	$1/\sqrt{2}$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\langle p \mid j \mid n\,\pi^+ \rangle$	1		
$\langle p \mid j \mid p\,\eta \rangle$	$1/\sqrt{6}$	$1/\sqrt{6}$	$1/\sqrt{6}$
$\langle p \mid j \mid \Lambda K^+ \rangle$	1		
$\langle n \mid j \mid n\pi^o \rangle$			$1/\sqrt{2}$
$\langle n \mid j \mid p\pi^- \rangle$		1	
$\langle n \mid j \mid n\eta \rangle$			$1/\sqrt{6}$
$\langle n \mid j \mid \Lambda K^o \rangle$			

REFERENCES.

(1) - M. Gell-Mann, Caltech Report CTSL-20 (1961)

(2) - Y. Ne'eman, Nucl. Phys. 26, 222 (1961)

(3) - R. E Behrends, J. Dreitlein, C. Fronsdal and B. W. Lee, R. M. P. 34, 1 (1962), Hereinafter called BDFL.